# An Ab Initio Study of the Diadic Prototropic Tautomerism $\mathrm{H}_{3} \mathrm{PX} \rightleftarrows \mathrm{H}_{2} \mathrm{PXH}$ ( $\mathrm{X}=\mathbf{O}, \mathbf{N H}, \mathrm{CH}_{2}$ ) 

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The diadic prototropic tautomerisms of three model systems $\mathrm{H}_{3} \mathrm{PX} \rightleftarrows \mathrm{H}_{2} \mathrm{PXH}\left(\mathrm{X}=\mathrm{O}, \mathrm{NH}, \mathrm{CH}_{2}\right)$ are investigated by $a b$ initio methods. Stationary points corresponding to local minima and transition states are located using gradient techniques at $\mathrm{HF} / 6-31 \mathrm{G}^{* *}$ level, and relative energies calculated at MP4/6-31G** including a zero-point energy (ZPE) correction, using the optimised geometries. Computation of vibrational frequencies shows a correlation between the norm of the transition-state imaginary frequencies and both the free energy of activation for hydrogen migration and the $H_{m}-P-X$ angle. The intrinsic barriers $\Delta G_{o}^{\ddagger}$ calculated by the Marcus theory indicate that all three unimolecular hydrogen transfers belong to a homogenous series. The disparity between the calculated (gas-phase) stabilities and experimental observations based on solution studies is noted. However, it is shown that consideration of the computed net charges dipole moments and electric field gradients of the isolated species, in conjunction with a simple perturbation expression for the solvation energy, allows a successful rationalisation of tautomeric ratios measured in solution.

Owing to the ease with which the phosphorus atom can change its valency from 3 to 4 , the diadic prototropic tautomerism (I) $\rightleftarrows$ (II) is in some cases a reversible process, the equilibrium position strongly dependent upon the nature of the substituents on both $P$ and $X$. Although closely related, experimental information about these processes varies. The existence of the methylenephosphorane-alkylphosphine ( $\mathrm{X}=$ $\mathrm{CR}_{2}$ ) equilibrium has only recently been established ${ }^{1}$ while the alkyl phosphonate-hydroxyphosphine ( $\mathrm{X}=\mathrm{O}$ ) rearrangement has been known in the chemistry of organophosphorus compounds for nearly 80 years, ${ }^{2}$ and the phosphimideaminophosphine ( $\mathrm{X}=\mathrm{NR}$ ) interconversion for more than three decades. ${ }^{3}$ Recent advances in the chemistry of these tautomerisms involving phosphorus atoms have been described in a comprehensive review by Mastryukova and Kabachink. ${ }^{4}$ Although there have been several detailed calculations on the electronic structure and chemical bonding in prototype phosphonium ylides ${ }^{5}\left(\mathrm{X}=\mathrm{CH}_{2}\right)$ and phosphine oxides $(\mathrm{X}=\mathrm{O})$ and their trivalent phosphorus isomers, little is known about the third member of the family, namely the phosphimide (iminophos- $\lambda^{5}$-phane) $\mathrm{H}_{3} \mathrm{P}=\mathrm{NH}(X=\mathrm{NH})$. Moreover, in previous studies, ${ }^{5.6}$ the interconversion pathway between quadri- and tri-valent isomers have not been fully discussed from the standpoint of the diad prototropic tautomerism. This lack of theoretical information for the $>$ PNR system prompts us to consider the rearrangement between model species $\mathrm{H}_{3} \mathrm{P}-\mathrm{NH} \rightleftarrows \mathrm{H}_{2} \mathrm{PNH}_{2}$. In addition, the large difference in the behaviour of $\mathrm{P}-\mathrm{C}, \mathrm{P}-\mathrm{N}$, and $\mathrm{P}-\mathrm{O}$ compounds with regard to the equilibrium (I) $\rightleftarrows$ (II) in solution suggests the need for a comparative study of the interconversions of these systems. The main purpose of the present study is thus to compare the unimolecular 1,2-hydrogen shifts connecting the prototype (I) and (II) species by performing calculations at the same high level of accuracy.

## Calculations

All calculations were performed using the Gaussian-81 ${ }^{7}$ program and two basis sets: 3-21G* (including only six cartesian $d$-functions on P ) ${ }^{8}$ and $6-31 \mathrm{G}^{*}$ (including polarisation $d$-functions on $\mathrm{C}, \mathrm{N}, \mathrm{O}$, and P and $p$-functions on H$).{ }^{9}$ The relative energies of the stationary points of each system were

estimated using Møller-Plesset perturbation theory to fourthorder, including all single, double, and quadruple excitations (MP4SDQ) ${ }^{10}$ at the $3-21 G^{*}$-optimized geometries with the HF/6-31G** wavefunctions as references for generating excited configurations. Harmonic vibrational frequencies were computed with the $3-21 \mathrm{G}^{*}$ basis making use of the analytic second derivatives. The best estimation for the energy differences between stationary points thus includes MP4SDQ/6-31G** values plus zero-point energy contributions.

## Results and Discussion

(1) The Phosphine Oxide-Phosphinous Acid Tautomerism.The pathway of the reaction $\mathrm{H}_{3} \mathrm{PO} \rightleftarrows \mathrm{H}_{2} \mathrm{POH}$ has previously been studied in some detail by Schmidt et al. ${ }^{6}$ These authors reported the optimised geometries of three stationary points (the two local minima and the transition state for 1,2shift connecting them) with the $3-21 \mathrm{G}^{*}$ basis set, and the energy differences between them at the MP3/6-31G* level. An energylocalised orbital analysis has also been described.

Table 1 contains total and relative energies and some thermochemical properties of $\mathrm{H}_{3} \mathrm{PO}(1), \mathrm{H}_{2} \mathrm{POH}(3)$, and the transition structure (TS) (2) calculated at different levels. Thermochemical properties relating to the gas-phase rearrangement are listed in Table 2.

Although the HF/3-21G* energies obtained for these species in the present work are lower ( $0.06 \mathrm{a} . \mathrm{u}$.) than those reported in ref. 6, calculated using five spherical harmonic $d$-orbitals on phosphorus, their optimised geometrical parameters are similar, and are not shown here. The TS (2) possesses $C_{s}$ symmetry; the migration of hydrogen occurs in the molecular plane. To see more clearly the type of motion involved in each normal mode during the transformation, harmonic vibrational frequencies are drawn as a correlation diagram in Figure 1. At

Table 1. Total (a.u.), relative (in parentheses, $\mathrm{kcal} \mathrm{mol}^{-1}$ ), and zero-point vibrational ( $\mathrm{ZPE} / \mathrm{kcal} \mathrm{mol}^{-1}$ ) energies, entropies ( $\mathrm{S} / \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ ), and specific heat capacity $\left(C_{\mathrm{p}} / \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}\right)$ for three stationary points on the $\mathrm{H}_{3} \mathrm{PO}$ energy surface ${ }^{a}$

|  | $\mathrm{H}_{3} \mathrm{PO}(\mathbf{1})\left(C_{3 v}\right)$ | $\mathrm{TS}(2)\left(C_{s}\right)$ | $\mathrm{H}_{2} \mathrm{POH}(3)\left(C_{s}\right)$ |
| :--- | :---: | :---: | :---: |
| $\mathrm{HF} / 3-21 \mathrm{G}^{* b}$ | $-415.26604(6.1)$ | $-415.12818(92.6)$ | $-415.27576(0.0)$ |
| $\mathrm{HF} / 6-31 \mathrm{G}^{* *}$ | $-415.31304(7.1)$ | $-417.18400(88.1)$ | $-417.32433(0.0)$ |
| MP4SDQ/6-31G** | $-417.64186(5.0)$ | $-417.53545(71.8)$ | $-417.64986(0.0)$ |
| ZPE | 21.6 | 18.0 | 21.2 |
| $S_{2}(298.15 \mathrm{~K})$ | 57.8 | 58.4 | 59.7 |
| $C_{\mathrm{P}}(298.15 \mathrm{~K})$ | 9.4 | 10.4 | 11.2 |

${ }^{a}$ Using the 3-21G* optimised geometry. ${ }^{b}$ The HF/3-21G* energies reported in ref. 6: (1) -415.207 01; (2) -415.068 22; and (3) -415.216 65 a.u.

Table 2. Thermochemical properties of the gas-phase $\mathrm{H}_{3} \mathrm{PO}(1) \longrightarrow$ $\mathrm{H}_{2} \mathrm{POH}$ (3) rearrangement

| Property $^{a}$ |  | Estimated |
| :--- | :--- | ---: |
| value |  |  |

${ }^{a} \Delta H(0 \mathrm{~K})=\Delta E(\mathrm{MP} 4)-\Delta \mathrm{ZPE} . \Delta H(298.15 \mathrm{~K})=\Delta H(0 \mathrm{~K})+$ $\int \Delta C_{\mathrm{p}} \mathrm{d} t . \Delta S_{12}=S_{2}-S_{1} . \Delta G=\Delta H-T \Delta S$.

Figure 1. Vibrational frequencies correlation diagram for the [ $\mathrm{H}_{3} \mathrm{PO}$ ] stationary points (3-21G*). Values are given in $\mathrm{cm}^{-1}$ and the scale is arbitrary
the TS (2), there is an overall shift to lower frequencies with respect to those in $\mathrm{H}_{3} \mathrm{PO}$. The stretching frequency in the TS is calculated to lie nearer to that of $\mathrm{H}_{2} \mathrm{POH}$ although the bending


$$
H^{\prime} P N X=85.8 \quad H N P X=202.6
$$



HPNH ${ }^{\prime}= \pm 117.8$
(4) $\left(C_{5}\right)$


HPNH $^{\prime}= \pm 131 \cdot 4$
(6) $\left(C_{s}\right)$


HPNX $=203.8 \quad H_{m} P N X=-60.1$
(5)


Figure 3. Vibrational frequencies correlation diagram for three [ $\left.\mathrm{H}_{4} \mathrm{PN}\right]$ stationary points. Values are in $\mathrm{cm}^{-1}$ and the scale is arbitrary

Table 3. Total (a.u.), relative (in parentheses, $\mathrm{kcal} \mathrm{mol}^{-1}$ ) and zero-point vibrational ( $\mathrm{ZPE} / \mathrm{kcal} \mathrm{mol}^{-1}$ ) energies, entropies ( $\mathrm{S} / \mathrm{cal}_{\mathrm{mol}}{ }^{-1} \mathrm{~K}^{-1}$ ), and specific heat capacities ( $C_{\mathrm{p}} / \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ ) for four stationary points on the [ $\mathrm{H}_{4} \mathrm{PN}$ ] energy surface

|  | $\mathrm{H}_{3} \mathrm{P}-\mathrm{NH}(\mathbf{4})$ | $\mathrm{H}_{2} \mathrm{P}(\mathrm{H}) \mathrm{NH}(\mathbf{5})$ | $\mathrm{H}_{2} \mathrm{P}-\mathrm{NH}_{2}(\mathbf{6})$ | $\mathrm{H}_{2} \mathrm{P}(\mathrm{H}) \mathrm{NH}(7)$ |
| :--- | :---: | :---: | :---: | :---: |
| HF/3-21G* | $-395.49170(40.8)$ | $-395.37839(111.9)$ | $-395.55669(0.0)$ | $-395.37606(113.3)$ |
| HF/6-31G |  |  |  |  |
| MP4SDQ/6-31G*** | $-397.43640(35.5)$ | $-397.32839(103.3)$ | $-397.49296(0.0)$ | $-397.32472(105.5)$ |
| ZPE | $-397.76102(33.4)$ | $-397.67406(88.0)$ | $-397.81434(0.0)$ | $-397.66731(92.2)$ |
| $S(298.15 \mathrm{~K})$ | 28.3 | 25.4 | 29.7 | 24.5 |
| $C_{\mathrm{p}}(298.15 \mathrm{~K})$ | 59.2 | 58.9 | 60.0 | 58.5 |
|  | 11.8 | 11.3 | 12.3 | 10.3 |

Table 4. Thermochemical properties of the gas-phase $\mathrm{H}_{3} \mathrm{PNH}(4) \longrightarrow$ $\mathrm{H}_{2} \mathrm{PNH}_{2}$ (6) rearrangement

|  |  | Estimated |
| :--- | :--- | ---: |
| Property ${ }^{a}$ | value |  |
| Heat of reaction $\left(\mathrm{kcal} \mathrm{mol}^{-1}\right)$ | $\Delta H_{46}^{\circ}(0 \mathrm{~K})$ | -32.0 |
|  | $\Delta H_{46}(298.15 \mathrm{~K})$ | -31.6 |
| Entropy of reaction $\left(\mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}\right)$ | $\Delta S_{46}(298.15 \mathrm{~K})$ | 0.8 |
| Free energy of reaction $\left(\mathrm{kcal} \mathrm{mol}^{-1}\right)$ | $\Delta G_{46}(298.15 \mathrm{~K})$ | -31.8 |
| Energy barrier (kcal $\left.\mathrm{mol}^{-1}\right)$ | $\Delta H^{\ddagger}(0 \mathrm{~K})$ | 51.7 |
|  | $\Delta H^{46}(298.15 \mathrm{~K})$ | 51.6 |
| Activation entropy (cal $\left.\mathrm{mol}^{-1} \mathrm{~K}^{-1}\right)$ | $\Delta S^{\ddagger}{ }_{46}(298.15 \mathrm{~K})$ | -0.3 |
| Free energy of activation | $\Delta G^{\ddagger}{ }_{46}(298.15 \mathrm{~K})$ | 51.7 |
| (kcal mol ${ }^{-1}$ ) |  |  |

the activation energy becomes smaller than that in the $\mathrm{H}_{3} \mathrm{PO}$ case. In fact, the free energy of activation for the gas-phase rearrangement (4) $\longrightarrow(6)$ via the TS $(5)$ is estimated at 51.7 $\mathrm{kcal} \mathrm{mol}^{-1}$. The activation entropy is negative in this reaction but its absolute value is negligible (see Table 4).

The geometry of aminophosphine (6) has been studied in some previous papers. ${ }^{11.12}$ Owing to the lack of polarisation functions on nitrogen atom in the $3-21 \mathrm{G}^{*}$ basis set, the amino group is found to be planar. Nevertheless, the energy difference between the planar and pyramidal forms has been shown to be extremely small ( $0.2 \mathrm{kcal} \mathrm{mol}^{-1}$ ). ${ }^{10}$

The $\mathrm{P}-\mathrm{N}$ bond length in the non-classical $\mathrm{H}_{3} \mathrm{PNH}$ (4) (1.525 $\AA$ ) is much shorter than that in $\mathrm{H}_{2} \mathrm{PNH}_{2}(1.678 \AA)$, but close to the experimental ( $X$-ray) value of $1.536 \AA$ determined for a substituted $>\mathrm{P}-\mathrm{N}$ - compound. ${ }^{13}$ It is noted that in this

Table 5. Total (a.u.), relative (in parentheses, $\mathrm{kcal} \mathrm{mol}^{-1}$ ) and zero-point vibrational ( $\mathrm{ZPE} / \mathrm{kcal} \mathrm{mol}{ }^{-1}$ ) energies, entropies ( $S / \mathrm{cal}^{\mathrm{mol}}{ }^{-1} \mathrm{~K}^{-1}$ ), and specific heat capacities ( $C_{\mathrm{p}} /$ cal $\mathrm{mol}^{-1} \mathrm{~K}^{-1}$ ) for four stationary points on the [ $\mathrm{H}_{5} \mathrm{PC}$ ] energy surface

|  | $\mathrm{H}_{3} \mathrm{P}_{-} \mathrm{CH}_{2}(\mathbf{8})$ | $\mathrm{H}_{2} \mathrm{P}(\mathbf{H}) \mathrm{CH}_{2}(\mathbf{1 1 )}$ | $\mathrm{H}_{2} \mathrm{PCH}_{3}(\mathbf{1 0 )}$ | $\mathrm{H}_{2} \mathrm{P}(\mathrm{H}) \mathrm{CH}_{2}(\mathbf{9})$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{HF} / 3-21 \mathrm{G}^{*}$ | $-379.54937(57.6)$ | $-379.43134(131.6)$ | $-379.64109(0.0)$ | $379.46705(109.2)$ |
| $\mathrm{HF} / 6-31 \mathrm{G}^{* *}$ | $-381.40212(58.5)$ | $-381.29297(127.0)$ | $-381.49538(0.0)$ | $-381.32307(108.1)$ |
| MP4SDQ/6-31G** | $-381.70698(58.7)$ | $-381.61115(118.8)$ | $-381.80042(0.0)$ | $-381.65073(94.0)$ |
| ZPE $(298.15 \mathrm{~K})$ | 35.1 | 31.3 | 36.9 | 33.4 |
| $S(298.15 \mathrm{~K})$ | 61.1 | 59.9 | 61.1 | 59.5 |
| $C_{\mathrm{p}}(298.15 \mathrm{~K})$ | 43.2 | 12.0 | 11.6 | 11.8 |

$\mathrm{HCPH}^{\prime}= \pm 83.7$
$\mathrm{HmPCX}=-71 \cdot 8$

$H P C H^{\prime}= \pm 120.5$
$H P C X=94.2 \quad H C P X=19.1$
(8) $\left(C_{s}\right)$
$H^{\prime} P C X=205 \cdot 3 \quad H^{\prime \prime} C P X=208 \cdot 6$
(9)


$H P C H^{\prime}= \pm 47.9$
$\mathrm{HPCH}_{m}= \pm 119.1$
$\mathrm{HCPH}_{m}= \pm 111 \cdot 2$
(10) $\left(C_{s}\right)$
(II) $\left(C_{s}\right)$

Figure 4. HF/3-21G* geometries of four stationary points on the [ $\mathrm{H}_{5} \mathrm{CP}$ ] energy surfaces. Bond lengths in $\AA$ and bond angles in degrees
crystal structure the phosphorus atom has a nearly planar conformation whereas the group in (4) is strongly 'pyramidal'. The calculated $\mathrm{P}-\mathrm{N}$ stretching frequency of $1294 \mathrm{~cm}^{-1}$ in (4) also ranges within the i.r. values of $1100-1495 \mathrm{~cm}^{-1}$ observed for a large number of phosphimide compounds. ${ }^{14}$ It seems that this frequency is quite sensitive to substituent effects, in particular by $\pi$-donor groups on the phosphorus atom. Such a frequency is close to that of the typical double bond of ( 1250 $\mathrm{cm}^{-1}$ ) in iminophosphane ( $\left.-\mathrm{P}=\mathrm{N}-\right)^{15}$

As in the $\mathrm{H}_{3} \mathrm{PO} \rightleftarrows \mathrm{H}_{2} \mathrm{POH}$ tautomerism, the PN stretching frequency in the transition state (5) is quite close to that of the product (6). The variation of the remaining normal modes also follows the pattern discussed for the phosphineoxide tautomerism. The imaginary frequency corresponding to the hydrogen shift is at $2131 \mathrm{i} \mathrm{cm}{ }^{-1}$, slightly smaller than the $\mathrm{H}_{2} \mathrm{P}(\mathrm{H}) \mathrm{O}$ value of 2359 i . Structure (7), which possesses $C_{s}$ symmetry, exhibits two imaginary frequencies. The first of them 2379 i is somewhat higher than the corresponding value in (5), whereas the second frequency of $515 \mathrm{i} \mathrm{cm}{ }^{-1}$ involves the out-ofplane movement of the migrating hydrogen. $\dagger$ The energy

[^0]Table 6. The predicted thermochemical data for the gas-phase $\mathrm{H}_{3} \mathrm{P}-\mathrm{CH}_{2}(8) \longrightarrow \mathrm{H}_{2} \mathrm{P}-\mathrm{CH}_{3}(10)$ rearrangement

|  | Parameters $^{a}$ | Predicted value |
| :--- | :--- | :---: |
| Heat of reaction | $\Delta H_{810}^{\circ}(0 \mathrm{~K})$ | -56.9 |
|  | $\Delta H_{810}(298.15 \mathrm{~K})$ | -57.4 |
| Entropy of reaction | $\left.\Delta S_{810} 298.15 \mathrm{~K}\right)$ | 0.0 |
| Free energy of reaction | $\Delta G_{810}(298.15 \mathrm{~K})$ | -57.4 |
| Energy barrier | $\Delta H_{810}^{ \pm}(0 \mathrm{~K})$ | 33.6 |
|  | $\Delta H_{810}^{!}(298.15 \mathrm{~K})$ | 33.2 |
| Activation entropy | $\Delta S_{810}^{!}(298.15 \mathrm{~K})$ | -1.6 |
| Free energy of activation | $\Delta G_{810}^{!}(298.15 \mathrm{~K})$ | 33.7 |
| ${ }^{a}$ See Table 1 for definition and units. |  |  |

difference between the two maxima is small $\left[\Delta\left(\Delta H_{298.15}\right) 3 \mathrm{kcal}\right.$ $\left.\mathrm{mol}^{-1}\right]$. This reflects the fact that the energy surface is extremely flat along the rotation of the $\mathrm{P}-\mathrm{H}_{\mathrm{m}}$ group around the $\mathrm{P}-\mathrm{N}$ bond. From a technical point of view, only the use of analytical energy second derivatives in the optimisation allows an unambiguous location of the true transition state. With regard to the preference of the non-symmetrical migration via (5) over the symmetrical one via (7), we refer to a detailed examination in an earlier paper. ${ }^{15}$
(3) The Methylenephosphorane-Methylphosphine Tautomer-ism.-The geometry of four stationary points (8)-(11) on the $\mathrm{CH}_{5} \mathrm{P}$ energy surface are shown in Figure 4. The computed energetic and thermochemical data are listed in Tables 5 and 6, respectively. Harmonic vibrational frequencies are collected in Figure 5.

The electronic and structural aspects of both potential minima methylphosphorane (8) and methylphosphine (10) have been discussed by a number of authors. ${ }^{16-18}$ The present 3$21 \mathrm{G}^{*}$-optimised geometries (Figure 4) are similar to earlier data (see ref. 18 for instance). The $\Delta H^{\circ}(0 \mathrm{~K})$ values estimated at MP3/6-1G** level in ref. 6 differ only slightly from the MP4SDQ/6-31G** data shown in Table 6.

For the conversion (8) $\longrightarrow(10)$, via the TS (9), our calculation predicts $\Delta G(298.15 \mathrm{~K})-57.4$ and $\Delta G^{\ddagger}(298.15 \mathrm{~K})$ $33.7 \mathrm{kcal} \mathrm{mol}^{-1}$ for the free energies of reaction and activation, respectively.

As seen in Figure 5, the $\mathrm{P}-\mathrm{C}$ and $\mathrm{P}-\mathrm{H}$ bonding of the phospholium ylide (8) vibrate at 1060 and $2542-2674 \mathrm{~cm}^{-1}$, respectively ( $3-21 \mathrm{G}^{*}$ ). These theoretical values are comparable with the experimental ones of 1078-1 085 and 2360-2 400 $\mathrm{cm}^{-1}$ obtained from i.r. spectra of a few substituted P-H ylides. ${ }^{19}$ It is well established that the $3-21 \mathrm{G} *$ frequency for the PH stretching mode is overestimated by $c a .10 \%{ }^{20}$ but the situation for the $\mathrm{P}-\mathrm{C}$ bond is not so clearcut. Like the previous case ( $\mathrm{P}-\mathrm{N}$ ) the latter appears to be highly susceptible to the presence of substituents. For example, the present $\mathrm{P}-\mathrm{C}$ frequency of $1060 \mathrm{~cm}^{-1}$ is markedly smaller than that of 1428 $\mathrm{cm}^{-1}$ previously calculated with a DZP basis set for the phospholium cyclopropyl ylide $\mathrm{H}_{3} \mathrm{P}=\mathrm{C}\left(\mathrm{CH}_{2}\right)_{2} .{ }^{21}$


Figure 5. Correlation diagram of the harmonic vibrational frequencies ( $\mathrm{cm}^{-1}$ ) for three stationary points on the $\mathrm{H}_{3} \mathrm{PCH}_{2} \longrightarrow \mathrm{H}_{2} \mathrm{P}-\mathrm{CH}_{3}$ rearrangement pathway. The scale is arbitrary


Figure 6. Energy profiles of three diadic prototropic tautomerisms $\mathrm{H}_{3} \mathrm{PX} \longrightarrow \mathrm{H}_{2} \mathrm{PXH}$

It is worth noting that unlike the $\mathrm{H}_{2} \mathrm{P}(\mathrm{H}) \mathrm{C}$ and $\mathrm{H}_{2} \mathrm{P}(\mathrm{H}) \mathrm{NH}$ transition states, the PC stretching frequency in the TS (9) ( $1036 \mathrm{~cm}^{-1}$, Figure 5) is much closer to that in the ylide form (8) $\left(1060 \mathrm{~cm}^{-1}\right)$ than to that in the phosphine form $(10)\left(737 \mathrm{~cm}^{-1}\right)$.

In addition, the imaginary frequency 1492 i in (8) is also significantly smaller than those of 2359 i in $\mathrm{H}_{3} \mathrm{PO}(3)$ and 2131 i in $\mathrm{H}_{2} \mathrm{PNH}$ (5). This is no doubt a reflection in the vibrational parameters, of the increased exothermicity of the rearrangement $\mathrm{H}_{3} \mathrm{PCH}_{2}(8) \longrightarrow\left(\mathrm{H}_{2} \mathrm{PCH}_{3}\right)$ (10) with respect to the earlier ones. The planar stationary point (11) has two imaginary frequencies of 3156 i and $786 \mathrm{i} \mathrm{cm}{ }^{-1}$. Both are large in comparison with the corresponding values in (9) (1492i for the former) and in (7) ( 515 i for the latter). Moreover, the energy difference between both maxima (9) and (11) is substantial (22.7 $\mathrm{kcal} \mathrm{mol}^{-1}$ at MP4 and ZPE, Table 5). Although the energy surfaces in the immediate neighbourhood of the oxide point (9) is also very flat along the co-ordinate describing the rotation of the migrating hydrogen around $\mathrm{P}-\mathrm{C}$ bond, a difference of this magnitude indicates on the one hand a clear preference for the non-symmetrical pathway over the symmetrical one and, on the other hand, the strong energetic compensation provided by the avoided repulsion between two migrating lone pairs [the $\pi(\mathrm{PH})$ and the carbon lone pair] for the disfavoured geometrical conformation of the TS (9).
(4) Summary and Comparison with Experiment.-The free energy profiles of the unimolecular prototropic tautomerisms considered are shown schematically in Figure 6. It is seen that the exothermicity of the reactions is appreciably increased in the sequence $\mathrm{H}_{3} \mathrm{PO} \longrightarrow \mathrm{H}_{3} \mathrm{PNH} \longrightarrow \mathrm{H}_{3} \mathrm{PCH}_{2}$. The successive replacement of O by NH and NH by $\mathrm{CH}_{2}$ thus increases the PH
acidity and stabilises the trivalent phosphine form by approximately the same amount (a free energy change of $c a .26$ $\mathrm{kcal} \mathrm{mol}{ }^{-1}$ ). Conversely, the free energy of activation decreases more rapidly in going from $\mathrm{H}_{3} \mathrm{PNH}$ to $\mathrm{H}_{3} \mathrm{PCH}_{2}\left(18 \mathrm{kcal} \mathrm{mol}^{-1}\right)$ than from $\mathrm{H}_{3} \mathrm{PO}$ to $\mathrm{H}_{3} \mathrm{PNH}\left(11.6 \mathrm{kcal} \mathrm{mol}^{-1}\right)$. This is in line with considerations based on Marcus' theory, ${ }^{22}$ which relates the free energy of activation $\left(\Delta G^{\ddagger}\right)$ to the free energy of the reaction ( $\Delta G$ ) by relationship (1) ${ }^{23}$ where $\Delta G^{\ddagger}$ represents the

$$
\begin{equation*}
\Delta G^{\ddagger}=\Delta G_{0}^{\ddagger}\left(1+\frac{\Delta G}{4 \Delta G_{0}^{\ddagger}}\right)^{2} \tag{1}
\end{equation*}
$$

'intrinsic barrier' ( $\Delta G_{\mathrm{o}}^{\ddagger}=\Delta G^{\ddagger}$ when $\Delta G=0$ ). Regarding the three 1,2 -shift studies as a series of homogenous reactions, we can now introduce the Brønsted exponent $\alpha$ which is defined by differentiating equation (1) leading to (2). Employing energetic

$$
\begin{equation*}
\alpha=\frac{\mathrm{d} \Delta G^{\ddagger}}{\mathrm{d} \Delta G}=\frac{1}{2}\left(1+\frac{\Delta G}{4 \Delta G_{0}^{\ddagger}}\right) \tag{2}
\end{equation*}
$$

data given in Figure 6 with the aid of equations (1) and (2), the kinetic parameters $\alpha$ and $\Delta G_{0}^{\ddagger}$ for each rearrangement were calculated and are listed in Table 7.
The calculated intrinsic barrier is found to be nearly the same in each case and amounts to $\Delta G_{o}^{\ddagger} 63 \pm 4 \mathrm{kcal} \mathrm{mol}^{-1}$. This supports the hypothesis that all three 1,2 -hydrogen-transfer reactions belong to a homogenous series.

It is well recognised that the norm of the imaginary frequency in the transition state $\left(v_{i}\right)$ indicates the height of the activation barrier. According to More O'Ferrall, ${ }^{24}$ this is also a measure of the transfer angle during a proton-transfer process. Indeed, Figure 7 shows a parabolic relationship between $v_{i}$ and $\Delta G^{\ddagger}$ and

Table 7. Bond order at transition state ( $n_{\mathrm{TS}}$ ), Bronsted exponent ( $\alpha$ ), and intrinsic barrier ( $\Delta G_{o}^{\ddagger}$ ) of three rearrangements

| Reaction | $\alpha$ | $\Delta G_{0}^{\ddagger} / \mathrm{kcal} \mathrm{mol}^{-1}$ |
| :--- | :---: | :---: |
| $\mathrm{H}_{3} \mathrm{PO} \longrightarrow \mathrm{H}_{2} \mathrm{POH}_{3}$ | 0.49 | 66.0 |
| $\mathrm{H}_{3} \mathrm{PNH} \longrightarrow \mathrm{H}_{2} \mathrm{PNH}_{2}$ | 0.44 | 66.7 |
| $\mathrm{H}_{3} \mathrm{PCH}_{2} \longrightarrow \mathrm{H}_{2} \mathrm{PCH}_{3}$ | 0.39 | 58.9 |



Figure 7. Correlation between the imaginary frequency $v_{i}\left(\mathrm{~cm}^{-1}\right)$ and (a) the free energy of activation ( $\Delta G^{\ddagger}$ ) and (b) bond angle $\mathrm{H}_{\mathrm{m}}-\mathrm{P}-\mathrm{X}$ $(\beta)$ for three $\mathrm{H}_{3} \mathrm{PX} \rightleftarrows \mathrm{H}_{2} \mathrm{PXH}$ rearrangements
a quasi-linear relationship between $v_{i}$ and the bond angle $\beta=H_{m} P X\left(H_{m}=\right.$ transferred hydrogen, $X=O, N$, and $\left.C\right)$. It is interesting that an inverse relationship $\beta=f\left(v_{i}\right)$ has been found for the 1,5 -hydrogen shift. ${ }^{25}$

It is important to remember that the above discussion is only valid for the tautomerism process under ideal gas conditions. We now attempt to use these results to understand experimental findings for reactions occurring in solution.

Consider first the oxygen diad. As seen in Figure 6, both $\mathrm{H}_{3} \mathrm{PO}$ and $\mathrm{H}_{2} \mathrm{POH}$ species are quite stable in the gas phase with respect to the hydrogen migration. In contrast, the tautomeric equilibrium in solution has been found to be completely displaced towards the tetra-co-ordinated phosphorus ABPH-O isomer. Proof of the existence of the hydroxy form (ABP-OH) has been obtained in the case where A and B are strong $\sigma$-captor (such as $\left.\mathrm{CF}_{3}\right)^{26}$ or $\pi$-donor (OR or SR) ${ }^{27}$ groups. Thus it is clear that the oxygen diadic tautomerism is crucially dependent upon substituents and solvent effects, which both favour the $\mathrm{P}-\mathrm{H}$ isomer over the $\mathrm{O}-\mathrm{H}$.

Table 8 shows the positive net charge on the phosphorus atom is substantially reduced in the activated complex (2), while the atomic charge on the migrating hydrogen becomes markedly positive. Thus, the polarity of the PH bond $\left(\mathrm{P}^{+}-\mathrm{H}^{-}\right)$ is significantly decreased in going from $\mathrm{H}_{3} \mathrm{PO}$ (1) to the transition state (2) $\left(\mathrm{P}^{+}-\mathrm{H}^{+}\right)$. This view is supported by the computed electric field gradients of the hydrogen (Table 8). The $q_{z z}$ components indicate the relative ionic character of chemical bonds. Possessing a large $q_{z z}$ value, the $\mathrm{H}_{\mathrm{m}}$ atom in (3) is hence more positive than that in (1) and (2). Although the polarity of the PH and OH bonds cannot be directly compared, both the net charge and in particular the $q_{z z}$ value seem to indicate that the $\mathrm{H}(\mathrm{O})$ atom in (3) is much more acidic than the $\mathrm{H}(\mathrm{P})$ in (1). As a consequence, the hydrogen shift could become more difficult to achieve in the direction $(1) \longrightarrow(3)$, than that in the opposite direction (3) $\longrightarrow(1)$. Introduction of $\sigma$-captor substituents appears to enlarge the positive net charge on the $H_{m}$ atoms in (2), whereas $\pi$-donor groups reduce the positive net charge of the P atom. Both actions create a similar effect by rendering the $\mathrm{H}_{\mathrm{m}}$ atom more labile and hence favouring the formation of the hydroxy form (3).

In addition, the large dipole moment of (1) (Table 8) suggests that this species is even more strongly favoured by polar solvents at the expense of (3). We can estimate the solvation

$$
\begin{equation*}
\mathrm{SE}=A \cdot \frac{\sum_{i} C_{i \mathrm{~L}}{ }^{2}}{\varepsilon_{\mathrm{L}}-\varepsilon_{\mathrm{H}}} \tag{3}
\end{equation*}
$$

energies (SEs) in a simple way via relationship (3) ${ }^{28-30}$ where $A$ is a proportionality constant, $C_{i L}$ the LUMO coefficients, and $\varepsilon_{\mathrm{L}}$ and $\varepsilon_{\mathrm{H}}$ the energies of the LUMO and HOMOs, respectively. In the present case, the SE arises from the stabilising charge-


Figure 8. Frontier orbital energies (eV) of each stationary point on the $\mathrm{H}_{3} \mathrm{PX} \rightleftarrows \mathrm{H}_{2}$ PXH pathways

Table 8. Molecular properties of stationary points calculated at HF/6-31G** level

| Property | $\mathrm{H}_{3} \mathrm{PO} \rightleftarrows \mathrm{H}_{2} \mathrm{POH}$ |  |  | $\mathrm{H}_{3} \mathrm{PNH} \rightleftarrows \mathrm{H}_{2} \mathrm{PNH}_{2}$ |  |  | $\mathrm{H}_{3} \mathrm{PCH}_{2} \rightleftarrows \mathrm{H}_{2} \mathrm{PCH}_{3}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (8) | (9) | (10) |
| Net charge: P | 1.05 | 0.65 | 0.57 | 0.91 | 0.55 | 0.47 | 0.70 | 0.40 | 0.30 |
| $\mathrm{X}^{a}$ | -0.72 | -0.77 | -0.75 | -0.86 | -0.85 | 0.88 | -0.70 | -0.58 | -0.57 |
| $\mathrm{H}_{\mathrm{m}}{ }^{\text {b }}$ | -0.11 | 0.31 | 0.37 | -0.13 | 0.25 | 0.30 | -0.06 | +0.06 | 0.15 |
| H(P) | -0.11 | -0.09 | -0.10 | -0.07 | 0.11 | -0.10 | -0.14 | -0.10 | -0.07 |
| H(X) |  |  |  | 0.29 | 0.26 | 0.29 | 0.13 | 0.14 | 0.14 |
| Dipole moment $\mu^{c}$ | 4.2 | 3.4 | 1.5 | 2.9 | 2.3 | 1.61 | 2.4 | 0.42 | 1.37 |
| Overlap population $\mathrm{P}=\mathrm{X}^{a}$ | 1.10 | 0.25 | 0.42 | 1.24 | 0.26 | 0.54 | 1.24 | 0.70 | 0.60 |
| Electric field gradient ${ }^{\text {d }}{ }^{\text {a }} \times x \times\left(\mathrm{H}_{m}\right)$ | -0.09 | -0.17 | -0.21 | -0.09 | -0.14 | -0.27 | -0.09 | -0.11 | -0.16 |
| ${ }^{9} y y\left(\mathrm{H}_{\mathrm{m}}\right)$ | -0.09 | -0.05 | $-0.26$ | -0.10 | $-0.04$ | -0.18 | -0.08 | $-0.04$ | $-0.15$ |
| ${ }^{9} z z\left(\mathrm{H}_{\mathrm{m}}\right)$ | 0.18 | 0.22 | 0.47 | 0.19 | 0.18 | 0.45 | 0.17 | 0.15 | 0.31 |

transfer interaction between the HOMOs of the phosphorus species and the LUMOs of electron-captor (protic, for example) solvent molecules.

According to the HOMO energies given in Figure 8, equation (3) predicts an increasing order of SEs as follows: TS(2) > $\mathrm{H}_{2} \mathrm{POH}(3)>\mathrm{H}_{3} \mathrm{PO}$ (1). This means that protic solvents might favour the formation of the phosphine form (3).
To summarise, the charge distribution data in the model species (1)-(3) of the oxygen diadic tautomeric pathways allow us to understand the trends observed in solution. For the nitrogen diad ABPH-NR $\rightleftarrows \mathrm{ABP}-\mathrm{NHR}$, where A, B, and R are alkyl, aryl, or alkoxy groups, the tautomeric equilibria were found to be displaced under different solution conditions towards the formation of the trivalent phosphine form (ABP-NHR). Unlike the oxygen diad, PH isomers can be detected for a large number of $\mathrm{P}-\mathrm{N}$ compounds, however. Equilibria involving both PH and NH forms are also frequently observed. Considering the carbon diad ABPH-CR ${ }^{1} \mathrm{R}^{2} \rightleftarrows$ $A B P-C H R{ }^{1} R^{2}$, the tautomerism is quite reversible; the equilibrium position depending on the nature of the solvent, concentration, temperature, and substituents at the $P$ atoms. ${ }^{21}$ From Table 8, we note that the charge distribution in both the nitrogen or carbon diad systems (4)-(6) and (8)-(10) presents many similarities to that in oxygen diad discussed above. This implies that a similar substituents effect also exists for both the PN and PC rearrangements.

Actually, strong $\sigma$-captor or $\pi$-donor substituents attached on one centre always favours the migration of a hydrogen to the adjacent centre. ${ }^{4}$ In particular, the predominance of the phosphine forms in the absence of solvent ${ }^{19}$ (i.e. in the pure liquids) can also be understood by the increasing exothermicity and the decreasing free energy of activation of the rearrangement, which favours them (Figure 6). On the other hand, Kolodiazhnyi ${ }^{19}$ showed that the ratio of PH and CH forms in the carbon diadic tautomerism of a given P-C compound varies from 10:90 in the presence of cyclohexane solvent to $61: 39$ in chloroform. This correlates well with the larger dipole moment in all of the PH forms, in comparison with their XH isomers (Table 8). In addition, according to the HOMO energies displayed in Figure 8 and equation (3), the opposite trend to that in the oxygen diad appears to be observed for both nitrogen and carbon diads. In a protic solution (such as chloroform) the solvation energy due to the PH form, is larger than that of the XH form and consequently the solvated PH compounds become favoured over the solvated XH. This argument, which is based on a very simple perturbation approach [equation (3)] can thus endorse the solvent effect as stated above.

In conclusion, we present in this work an ab initio study on
the prototropic tautomerisms of three simple model diads. The behaviour of species considered in the gas phase appears to be quite different from that observed in solution. Nevertheless, a detailed analysis of calculated data yields a better understanding of these transformations which have been investigated experimentally over the past several decades, and provides in the same occasion a reasonable theoretical support for interpretations based on experimental findings. ${ }^{4}$

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[^0]:    * The remaining vibrational frequencies of structure (7) are (3-21G*) $3610,2562,2533\left(\mathrm{a}^{\prime \prime}\right), 2304,1337,1208,1054,933,843\left(\mathrm{a}^{\prime \prime}\right)$, and 786 cm ( $\mathrm{a}^{\prime \prime}$ ).

